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A Compendium of Views on the NCTM Standards

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Reactions to the publication of the National Council of Mathematics (NCTM) *Standards*, in both 1989 and 2000, revealed the extent to which differing sets of values and beliefs had divided the mathematical community. This paper rejoins the debate surrounding the *Standards*, exploring some of the opposing points of view while offering some perspectives on the roots of the conflict, the current status of the debate, and some suggestions as to what might be done to foster a shared vision for the future of mathematics education. The authors illustrate how various beliefs about the key issues of today's mathematics education reform initiative might cloud the way the *Standards* are interpreted and implemented and argue that ongoing debate over the *Standards* should be considered a constructive means of exploring possible avenues for the reconciliation of conflicting ideas while providing safeguards against both an erosion of the reform effort and desultory implementation of the *Standards*.

The publication of the NCTM (1989) *Standards* touched raw nerves and created an extensive dialogue within and among many groups whose interpretations articulated multiple perspectives on the condition and purpose of mathematics education. More than simple analyses of the issues from opposing points of view, discussions about the *Standards* are often complex, with important educational, philosophical, social, and political dimensions. It is interesting to note that while some criticized the *Standards* for being too extreme (Cheney, 1997; Finn, 1993; Haimo, 1998; Wu, 1997), others argued the need for more radical changes (Apple, 1992). One of the aims of this paper is to examine objectively how different individuals and groups perceive the NCTM's *Standards* (1989; 2000) and how these various perceptions might be consolidated into a cohesive force for change. The authors believe that what some may hear as a cacophony of voices can be experienced as a developing harmony if we are mindful of the contexts from which these various voices emerge and thereby understand the evolutionary process behind the publication of the *Standards*.

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A brief historical sketch

Over the second half of the twentieth century, the nature of mathematics education in the U.S. changed dramatically as it passed through reincarnations based on New Math, Back to Basics, Problem Solving, and the NCTM *Standards* (NCTM, 1989; 2000). The driving forces and educational objectives of each initiative were different, and analyzing or interpreting these reform movements requires a consideration of the social and political climate in which each reform was initiated.

The first reform movement, New Math, was on the drawing board even before the launch of Sputnik in 1957, but the Soviet Union's scientific achievement in space created in America's academic and political arenas a sense of immediacy about improving mathematics education in the United States. Policymakers perceived a need for a new generation of highly qualified mathematicians, scientists, and engineers whose work would produce, among other things, a space program which would outpace that of the Russians. The New Math reform promised to decrease the gap between university mathematics and high school mathematics (Howson, Keitel, & Kilpatrick, 1981). A de facto shortcoming of the movement was the creation of a curriculum based solely on logical principles, dismissing the psychological dimensions of learning; but even more damaging was the lack of effective professional development for teachers who were to implement the program (Bass, 1994). Consequently, the movement's ideals did not mesh well with classroom reality; and,

inevitably, there was a demand for changing the mathematics curriculum and instructional practices (Kilpatrick, 1992). The change came in the form of an almost complete reversal, a Back to Basics backlash.

In the 1970s, the Back to Basics movement was based on behavioral principles supported by the work of Thorndike and Skinner, and instructional strategies were focused on basic skills, emphasizing the mastery of computation (Howson, et al., 1981). Because teachers had been perceived as generally ill-equipped for the instructional demands of New Math, it was thought that well-designed instructional materials could overcome any shortcomings in teachers' content knowledge. What soon became evident in the Back to Basics movement was that there was no such a thing as a "teacher-proof" mathematics curriculum (Erlwanger, 1973), and the mathematical education community once again was faced with the challenge of developing a curriculum to bring effective mathematics instruction into the classroom.

In response to this need, NCTM initiated Problem Solving, an approach to mathematics instruction in which problem-solving techniques, within modified real-world contexts, would promote meaningful learning and teaching of mathematics (NCTM, 1980). Although such problems had always been an integral part of the mathematics curriculum, this initiative considered the process of problem solving a vehicle for learning mathematics by encouraging students to develop logical reasoning skills and take responsibility for their learning (Stanic & Kilpatrick, 1989).

Throughout the 1980s, the importance of raising expectations for students, providing mathematics within a historical context, and demonstrating the usefulness of mathematical understanding became the focus of various research efforts. Publications such as *A Nation at Risk* (National Commission on Excellence in Education, 1983) and *Educating Americans for the 21st Century* (National Science Board Commission, 1983) documented the need for change and generated recommendations for mathematics education reform. At the same time, mathematics education started to gain legitimacy with a proliferation of mathematics education research, produced by an ever-increasing number of scholars. With the emergence of constructivist theory as a dominant presence, there was a shift in mathematics education research from the investigation of teacher and student behaviors to an exploration of cognition and context (Cooney, 1994). The NCTM *Standards* (1989) was born in this interesting arena, where researchers were beginning to identify vital components of learning and teaching mathematics and in which remnants of New Math, Back to Basics, and Problem Solving were very much in evidence.

Policymakers also began to reaffirm the need for the educational sector to meet the needs of the economy (Glickman, 1998). Leaders of business and industry made their views known, especially as to the need for all students to be able to reason, design models, think creatively, and solve problems. There was a growing awareness that mathematics education was a part of the political structure and that mathematics educators could no longer be ignored during the formulation, debate, and implementation of policies and actions affecting mathematics instruction at all levels (Carl & Frye, 1991). With such empowerment, mathematics educators were motivated to better promote their discipline to both the public and policymakers (Crosswhite, 1990).

Concurrent with these developments, technological advances were being integrated into business and industry; and schools were expected to prepare students for the emerging information age. The utilization of computers was seen as a driving force for scientific and intellectual progress, with applications in most academic disciplines; and the new technologies contributed to the production of an expanding mass of mathematical knowledge (Bass, 2003; Hekimoglu, 2002). With this new capability came ontological and epistemological concerns and discussions about the process of mathematical proof (Bass, 2003; Ernest, 1998; Lakatos, 1998).

In the wake of the technological revolution, a quasi-empiricist and fallibilist philosophical stance based on the works of Gödel, Wittgenstein, and Lakatos emerged (Ernest, 1991). Mathematics should not be seen as a cornerstone of absolutism, especially in terms of its former identity as an objective and culture-free discipline. The existence of a multicultural society demanded awareness of differences in achievement in mathematics for African American, Hispanic, Native American, female, and low-income students (Moses, 1994; National Commission on Teaching and America's Future, 1996; National Research Council, 1989). Many believed a redefinition was in order and that "equity in mathematics education" should be based upon enrichment, fairness, empowerment, and cultural diversity.

In the 1980s, NCTM tried to inform the general public about the perceived crisis in education and called for a significant departure from then-current practice in terms of content and pedagogy. The goal was to help the mathematics education community

reach an accord with the needs of society and students, developments in mathematics and the application of mathematics, and the evolution of understanding about mathematics learning (NCTM, 1980; 1989). The 1989 *Standards,* designed to make mathematics accessible to all students, was startling to many mathematicians and mathematics educators, but it was primarily based on evidence uncovered by mathematics education researchers regarding what works, what does not work, and what could work more effectively and equitably (Hiebert, 1999; Research Advisory Committee, 1988).

What are the *Standards*? Who needs them?

Understanding and addressing concerns about the *Standards* requires a twofold approach: first, educators and mathematicians must agree that there is a need for having standards; and, second, they must have an awareness of, and an appreciation for, various perspectives on the issues addressed by the *Standards*. Among the initial concerns was what could, or could not, be inferred from the name of the publication. Generally, "standards" are used to establish measurable and ascertainable degrees of uniformity, accuracy, and excellence for products or services; and such "standards" are based on measurements, principles, and agreements which contain precise and static criteria, rules, and characteristics. Consequently, various objections were raised regarding the use of the term "standards" in the context of mathematics education (Les Steffe, in personal communication, 10-12-2003) because in the context of the NCTM publication, the term is equivocal and overly broad. Some questioned whether the NCTM *Standards* was merely a collection of slogans (Apple, 1992) or a well-defined statement about what mathematically literate students should know and be able to do: Was this document based on consistent philosophical and political stances about mathematics teaching and learning (Romberg, 1992; 1998)?

In the literature, individuals advocate using the *Standards* to reach an array of goals. Among those purposes: Elimination of vast differences in the quality of mathematics education (Delpit, 1995; Romberg, 1992); enhancing effectiveness of mathematics education by providing a clear focus for instruction, learning and assessment (Ravitch, 1995); demonstration of agreement and consensus (Labaree, 1984; O'Day & Smith, 1993); facilitation of the exchange of information (Noddings, 1992); establishment of a framework for school accountability (Apple, 1992; Labaree, 1984); confirmation of the legitimacy of mathematics education as a discipline by steering the profession towards making investments in better prepared teachers (Darling-Hammond, 2003; Labaree, 1984); strengthening mathematics education by bridging academic levels, including higher education; guidance in the writing of mathematics curricula; communication to the public and policymakers about what students should know and be able to accomplish (Falk, 2000); and development of a national curriculum in response to international comparison studies of countries whose students have achieved excellence in mathematics.

Fully accommodating each of these various issues in a compendium of mathematics standards would be virtually impossible. Such a document would need to be pragmatic yet comprehensive, broad yet focused and effective, grandiloquent yet succinct. Furthermore, beyond the array of issues to be addressed, there lies an overwhelming assemblage of opinions on those issues. Inevitably, the NCTM *Standards* (1989) was used for a variety of purposes, some of which were not in line with the developers' initial intentions of the publication being viewed as a resource guide (Romberg, 1992). From the developers' points of view, the primary aim of the *Standards* was to describe a vision in which mathematics education would promote mathematical thinking by creating an awareness of the nature of mathematics, its role in contemporary society, its cultural heritage, and the importance of mathematics as an instrument and tool of learning (Romberg, 1992; 1998). NCTM made a credible and admirable attempt with the 1989 *Standards*, but as one might have expected, an intense political and philosophical debate, dubbed the *math wars,* began with the publication of NCTM (1989) *Standards*.

Critiques on NCTM *Standards*

Appraisal of the NCTM *Standards* through bonafide and responsible critique depends on systemic considerations as well as specificity as to areas of interest, academic disciplines, and expectations. A comprehensive view of the NCTM *Standards* is needed to avoid focusing on any one of its components without regard to how that component fits into the overall scheme. It is not the authors' intent to suggest that the viewpoints of mathematicians and mathematics educators are inherently dichotomous, or that mathematics educators at different levels have oppositional agendas, but it does seem that each group has a tendency to lose sight of common goals. It is the authors' belief that understanding critiques of the *Standards* and using those critiques to initiate discourse about the issues may help build bridges

between the mathematics and mathematics education communities as well as define and address concerns within each group. These challenges can be successfully met if those involved in the debate will conform to a certain etiquette of criticism - articulating one's position is not mutually exclusive to making an effort to understand, appreciate, and respect the merits of an opposing point of view.

Of course, many criticisms grew, not from misinformation or misinterpretation, but from differences in the opinions and beliefs of well-informed mathematicians, mathematics educators, policymakers, parents, and others concerned about the future of mathematics education and how best to achieve an excellent educational environment. Some of these criticisms stemmed from interpretations about the purpose of decreased levels of attention or emphasis on particular skills in the curriculum without a clear understanding of the developers' initial message that the emphasis had to be shifted from students' proficiency at learning skills to their understanding of the mathematics underlying those skills (Romberg, 1992, 1998). Some criticisms were spawned by the *Standard's* challenge to the tacit traditions of mathematics instruction at all levels.

In the NCTM (1989) *Standards*, one reader might see a focus on the process of mathematics learning, another reader observes the big ideas of teaching mathematics, and still another may describe or interpret the same items in terms of instructional strategies. Many individuals expressed negative judgments about the *Standards* and the direction in which mathematics education was headed (Ewing, 1996; Haimo, 1998). In particular, the NCTM *Standards* (1989), was severely criticized for its recommendations for a reduced emphasis on arithmetical computation and symbolic-manipulation skills; the need for teaching formal proofs (Cheney, 1997; Finn, 1993; Haimo, 1998; Roitman, 1998; Wu, 1997); the use of multiple assessment strategies; the integration of technology into mathematics instruction (Weiss, 1992); the de-emphasis of the abstract in favor of the concrete; and an emphasis on cooperative learning (Cheney, 1997; Haimo, 1998; Roitman, 1998; Wu, 1997).

The vision statement kindled a national discussion about the nature of the very heart of mathematics education, the need for defining literacy in mathematics. Defining mathematical literacy is not only dependent on one's ontological beliefs about what mathematics is but also the epistemological beliefs regarding how it should be taught and the axiological beliefs about where and how it is used. As defined in the NCTM *Standards* (1989), one who has become mathematically literate has become confident in one's own ability to reason mathematically, has become a mathematical problem solver, and has learned to communicate mathematically. Following a decade of discussion and reflection, the 2000 NCTM *Standards* (2000) expands the definition of being mathematically literate to include having mathematical knowledge as a functional member of changing world: "Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem solving…." (p. 4).

What did the NCTM *Standards* (2000) say about skills and cooperative learning?

The NCTM *Standards* (1989) was criticized for advocating a reduction in the traditional emphasis on skills. The *Standards* message was that learning mathematics should not be limited to performing specific algebraic manipulation or basic arithmetic skills but should be expanded to include an in-depth understanding of concepts underlying these skills (NCTM, 1989; 2000). The debate on basic skills versus conceptual understanding goes back more than four decades (Brownell, 1956); and, although it might be reasonable to think that developing conceptual understanding might come at the expense of the development of basic mathematical skills (Roitman, 1998; Wu, 1997), the *Standards* aim was not to downplay the importance of basic skills. It was hoped that by providing students with an overall understanding of the role played by mathematics in their lives, students would be motivated to understand the mathematical concepts as well as master the skills. In NCTM *Standards* (2000), importance of basic skills in the curriculum was underscored by statements such as the following: "Fluency with basic addition and subtraction number combinations is a goal for pre-K-2 years" (p. 84); and "when students leave grade 5, they should be able to . . . efficiently recall or derive the basic number combinations for each operation" (p. 149).

From the debate about basic skills vs. conceptual understanding, it became clear such terms as "basic skills" are not universally defined by any stretch of the imagination. Therefore, when interpreting the *Standards* as well as critiques of the *Standards*, a critical question arises: when people talk about basic skills, or any other issue, are they talking about the same thing? And why isn't there more of a consensus about the meanings of these terms?

Prior to NCTM's Problem Solving initiative, basic skills were widely understood to be arithmetical computations and symbol manipulations. With advances in technology and access to global information and resources, "basic skills" have expanded to include data collection and analysis, measurement, and problem solving strategies; i.e., the use of logical reasoning and the application of basic algebraic and geometric concepts. The definition continues to be transformed, and there is little wonder that many criticisms have focused on the treatment of basic skills and their place in the curriculum.

Some might define working within a group as a basic skill. The *Standards*' support for the use of cooperative learning strategies was rooted in the developers' perception of classroom discourse as a requirement for the development of mathematical thinking and communication (NCTM, 1989; 2000). The primary goals of cooperative learning include enabling students to take responsibility for their learning, to develop mathematical judgment, to lessen reliance on outside authority, and to promote mathematical communication. Implementation of cooperative learning strategies in the mathematics curriculum reflects the fact that many academic disciplines expect students to work cooperatively (Hekimoglu, 2003a) and believe the process of doing mathematics is an important form of social interaction (Dowling, 1998; Ernest, 1998). Some educators justify the use of cooperative learning strategies by reference to the traditional inquiry by future employers concerning a prospect's ability to be a team player. The authors question, however, whether being a "team player" is in fact analogous to working effectively within a group and whether or not the overuse of cooperative learning strategies might inadvertently undermine the development of independent and individual learning styles. Participating in cooperative learning experiences might enable students to understand the mathematical aspects of society (Bishop, 1991), but the *Standards* did not suggest using it in every teaching period. Discussions about cooperative learning require the consideration of research findings that support *why and when and how* cooperative learning strategies should be implemented as well as pointing out possible pitfalls to avoid.

What did the NCTM *Standards* (2000) say about proof and technology?

Despite rumors to the contrary, the NCTM *Standards* (1989; 2000) never regarded mathematical proofs as outmoded or unimportant. Prior to publication of NCTM *Standards* (1989), mathematical proof in the mathematics curriculum had become either nonexistent or had receded into meaningless ritual, perhaps as a result of the Back to Basics emphasis on skills. Students not only entered collegiate level mathematics courses without having an appreciation for the importance of mathematical proof but also without the skills and knowledge required for proof construction (Schoenfeld, 1987; Wu, 1994). One goal of the *Standards* was to provide an impetus for having students develop an understanding of informal proof through the use of heuristic arguments and explanations about their conjectures with the hope this process would lead students to the development and appreciation of formal mathematical proof (NCTM, 1989).

To accomplish this goal, the *Standards* advocated the use of technological tools as a method for demonstrating mathematical ideas and to help students generate hypotheses. Contrary to common misinterpretations (Finn, 1993; Wu, 1997), the message was neither that the use of technological demonstrations should replace the need for proof nor that the construction of a mathematical proof should rule out the use of technology. The *Standard's* call for de-emphasizing formal mathematical proof was partly grounded in philosophical developments, based on the works of Gödel and Lakatos, questioning the historical gate-keeping role of mathematics, particularly the role of formal proof in the process of gaining acceptance by mathematicians (Ernest, 1991; Hersh, 1986). In response to the reactions and clarification about this issue stemming from the NCTM *Standards* (1989), the NCTM *Standards* (2000) clearly stated the need for doing mathematical proofs as a consistent part of students' mathematical experience: "By the end of secondary school, students should be able to understand and produce mathematical proofs" (p. 56).

What response does the mathematics community have to this recommendation? Mathematics education research has found that students do not share mathematicians' perceptions of doing proof as a backbone of doing mathematics and as way of doing mathematical research (Harel & Sowder, 1998; Selden & Selden, 2003). The necessity of proof for developing mathematical competency is not a universally accepted truth, and some math-based academic disciplines de-emphasize mathematical proof (Hekimoglu, 2003a). Additionally, studies have found that many pre-service teachers exhibit a skepticism about the relationship between proof and mathematical understanding (Hekimoglu, 2003b; Pandiscio, 2002). Furthermore,

where proof has been a traditional part of the curriculum, such as the use of two-column proofs presented in many high school geometry courses, there is an indication that the practice is not serving the educational purpose of teaching proofs (Herbst, 2002).

There seems to be a trend toward the use of informal and empirical arguments, as opposed to more formal mathematical proofs, in mathematical reasoning (Hekimoglu, 2003a; 2003b), indicating that formal proof remains of questionable value outside the mathematics research community. The inclusion of instruction about formal mathematical proof is often advocated as a necessity for helping students develop mathematical maturity; but exactly how doing mathematical proofs contributes to mathematical understanding is a relatively unexplored area. For many, successfully learning the process of formal proof construction requires an arsenal of previously developed logic skills rather than having one's reasoning skills developed by the process of learning to construct proofs. In the next revision of the *Standards*, it seems there is a need for clarification about the following critical questions: what is a proof, in what way is the process of constructing mathematical proofs an efficient way to teach logical reasoning, and how can mathematical proof be presented so as to engender a spirit of mathematical curiosity.

Another criticism of the NCTM *Standards* (1989) was centered on the didactic nature of technology integration. The *Standards* did not suggest that technology would be a magic bullet, correcting all the ills of the past or serving as a replacement for skill development (Romberg, 1992). The *Standards* position was based on the premise that technology was essential in the teaching and learning of mathematics and that technology influenced how and what mathematics might be taught as a result of the widespread impact of technology on society (Hansen, 1984; Kaput, 1992). In regard to the 2000 *Standards*, seeing technology simply as a tool for instruction would not only reflect a limited view of technology but would also undermine the power of technology available to both the public and mathematics education community. Furthermore, the commitment to the integration of technology into the curriculum at all levels was supported by mathematics education research findings suggesting technological tools were being used for a variety of different purposes ranging from computational assistance to intelligent tutorials to a medium for exploration and discovery.

It seems something of a mystery that many mathematicians object to the use of technology in mathematics instruction when technology is commonly used to conduct research and communicate findings. An inability to understand how technological tools can be used to solve mathematics problems often produces a dissatisfaction with students' mathematical achievement and their ability to use technological tools to solve problems in other disciplines (Hekimoglu, 2003a). With so many different messages being expressed by professors in method courses and mathematics classes about the value, or lack thereof, and the use, or misuse, of technology in mathematics education, pre-service teachers are being asked to negotiate a quagmire as they develop opinions and beliefs about teaching with technology (Leatham, 2002). Clearly, the use of technology in the mathematics curriculum is a complex issue involving the interplay of multiple factors, and clarification of these issues requires careful examination of research findings that shed light on the key components of successful technology integration.

And now what?

Perhaps the *Standards* greatest contribution to the mathematical community has been its role as a stimulus for more than twenty years of nonstop debate about these issues. During that time, virtually every aspect of mathematics education has been examined and discussed and debated in various arenas even beyond the confines of academia. Journalists and policymakers have raised a number of questions about mathematics, mathematics education, and the relationship between them. Mathematics educators are learning to communicate their position more clearly, especially on controversial issues such as basic skills, technology, and cooperative learning.

Philosophical and epistemological concerns about mathematics and mathematics education need to be publicly acknowledged. Constructivist pedagogy, for example, has often been misunderstood, perceived as a patchwork philosophical stance (Howe, 1998; Klein, 1997; Cheney, 1997). Mathematicians need to find ways to reconcile the inherent conflict between the formalist or Platonist view of the nature of mathematics (Ernest, 2000; Santucci, 2003; Schechter, 1998) and the constructivist pedagogy reflected in much of the *Standards*. Furthermore, beyond personal preferences and beliefs, society demands that *why, how,* and *what* mathematics should be taught depend on "simultaneous objective relevance and subjective irrelevance of school mathematics to one's set of values" (Ernest, 2000, p. 3).

Mathematicians and mathematics educators need to communicate with the general public, as well as with each other, about the primary goals of teaching mathematics and why these goals cannot be centered around the "high call" of promoting mathematical growth and/or creating mathematicians, although those are certainly worthy goals (Noddings, 1993; Woodrow, 1997). While mathematicians might express their desire that "students in schools should be taught abstract mathematical procedures through repeated practice of the procedures, in order that they reach the university conversant in the range of methods that they will need to use and apply there" (Boaler & Greeno, 2000, p.188), mathematics educators are keenly aware that many students will not be doing college level mathematics and that the diverse historical, social, philosophical, and psychological dimensions of mathematics learning and teaching cannot be ignored. In regard to this issue, educators need to articulate why secondary mathematics education cannot be based solely on preparing students for collegiate level when there is a perceived need to produce a critical future supply of highly qualified mathematicians (Lutzer & Maxwell, 2003). Still, it is important to help college-bound students make a smooth transition from high school to college, and more dialogue between mathematicians and mathematics educators is needed to connect high school and college mathematics curricula and instructional strategies (Adelman, 1999). The debates around the NCTM *Standards* might be beneficial for resolving conflicts about what students need to know to be prepared for college as well as what they need to know to become productive members of society.

Even though mathematics education research at the collegiate level is still in an embryonic stage, it has generally supported the *Standard's* stance on such issues as making connections between classroom mathematics and physical world applications to promote the development of conceptual understanding (Arney & Small, 2002; Karian, 1992; Solow, 1994). Moreover, the *Standard's* vision of technology has been re-enforced in the collegiate mathematics community by The Mathematical Association of America (MAA) with its presentations of research findings in documents such as *Computers and Mathematics* (Smith, Porter, Leinbach, & Wenger, 1988) and *The Laboratory Approach to Teaching Calculus* (Leinbach, Hundhausen, Ostebee, Senechal, & Small, 1991). The importance of implementing cooperative learning strategies in undergraduate mathematics classes has also been echoed in the *Cooperative Learning in Undergraduate Mathematics* (Rogers, Reynolds, Davidson, & Thomas, 2001) and *A Practical Guide to Cooperative Learning in Collegiate Mathematics* (Hagelgans et al., 1995).

The development of the next revision of the *Standards* demands that mathematics educators consider how the current provisions have actually been implemented (Roitman, 1998) and what overall systemic changes could be made to promote the success of the reform movement. Such a goal requires a collaboration between mathematicians and mathematics educators through discussion and delineation of the issues surrounding systemic changes in secondary education. Careful consideration of existent research findings on teaching and learning mathematics as well as the further development of specific research methods and theoretical frameworks, drawn from fields of study such as anthropology, sociology, psychology, linguistics, and philosophy may enable educators to question and explore the nature of mathematics, mathematics teaching and learning, and school structures.

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